Zero-bias conductance anomaly in bilayer quantum Hall systems[*]

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Bilayer quantum Hall system at total filling factor $\nu = 1$ shows a rich variety of broken symmetry ground states because of the competition between the interlayer and intralayer Coulomb interactions. When the layers are sufficiently close, a bilayer system develops spontaneous interlayer phase-coherence that manifests itself through a spectacular enhancement of the zero-bias interlayer tunneling conductance. We present a theory of this tunneling conductance anomaly, and show that the zero-bias conductance is proportional to the square of the *quasiparticle* tunneling amplitude.

Introduction: Bilayer systems consist of two 2D electron gases separated by a distance d comparable to the typical distance between electrons within one layer, and have been intensely investigated over the past decade [1]. Weakly disordered bilayer quantum Hall systems at total filling factor $\nu = 1$ undergo a quantum phase transition from a compressible state to a $\nu = 1$ quantum Hall state with spontaneous interlayer phase-coherence when the layers are sufficiently close [1]. It is conventional to use a pseudospin to represent the layer index, where pseudospin "up" denotes a state in the top layer, and pseudospin "down" denotes a state in the bottom layer. In the pseudospin language, the phase-coherent state can be viewed as an easy-plane ferromagnet with its pseudospin polarization $\vec{M}$ along the x-axis, $M_x = 1$. In analogy with ferromagnets, the low-lying excitations of a phase-coherent bilayer system are long-wavelength fluctuations of the ordered moment, *i.e.* pseudospin waves. These collective modes transfer charge from one layer to the other, and have a linear dispersion at long wavelengths when the interlayer tunneling amplitude Δ_t is zero [1]. The close similarity between the phenomenological effective theory of a phase-coherent bilayer system and that of a Josephson junction has led to suggestions that the phase-coherent bilayers should exhibit a dc Josephson effect in interlayer tunneling measurements [2–5].

The first *direct* experimental signature of the phase-coherent state and the linearly dispersing collective mode was observed by Spielman *et al.* who discovered a dramatic enhancement of the zero-bias tunneling conductance G_T as the layer separation d (measured in units of magnetic length) was reduced [6,7]. Although this dramatic enhancement is reminiscent of the tunneling characteristics of a Josephson junction, it is not yet clear whether the finite width and height of the peak is intrinsic, or is governed by the temperature and experimental limitations.

Here we present a theory of the zero-bias conductance G_T that is non-perturbative in the tunneling amplitude Δ_t. We find that the tunnel conductance G_T is given by Fermi's Golden rule for the *quasiparticles* in the phase-coherent state, and that the breakdown of Fermi's Golden rule for the bare electrons is therefore expected. In the following sections we first discuss the physical picture that underlies our theory, and then present the results of an approximate but fully microscopic calculation. The details of this calculation are available elsewhere [8].

Physical Picture: Let us consider a bilayer quantum Hall system at total filling factor $\nu = 1$ with tunneling amplitude Δ_t. In the pseudospin picture, the tunneling amplitude acts as a field along the x-axis in the pseudospin-space, whereas a bias-voltage or a random disorder potential acts as a field along the z-axis. In the mean-field approximation, the interlayer exchange Coulomb interaction enhances the splitting between symmetric and antisymmetric single-particle energies, $\Delta_{qp} = \Delta_t + M_0 \Delta_{sb}$, where M_0 is the dimensionless order-parameter for the phase-coherent state. This exchange-enhancement $M_0 \Delta_{sb}$ survives in the limit $\Delta_t \to 0$ and gives rise to spontaneous interlayer phase-coherence [1]. We will call the exchange-enhanced splitting Δ_{qp} as the quasiparticle tunneling amplitude.

The tunneling amplitude Δ_t is the only term in the microscopic Hamiltonian that does not conserve the charge in each layer separately. Recent theoretical work [5,9–11] has shown that the bare-electron Golden rule used to estimate the zero-bias conductance G_T breaks down as $\Delta_t \to 0$, *i.e.* that $\lim_{\Delta_t \to 0} G_T / \Delta_t^2$ diverges in the phase-coherent state. Evaluation of G_T therefore requires a non-perturbative theory like the one we present here. Our finding, that G_T is finite once the presence of low-energy quasiparticle excitations at finite temperature $T \neq 0$ and with disorder even at $T = 0$ is acknowledged, is at odds with the prediction that this system should show a dc Josephson effect. In usual dc Josephson effect systems, the persistent currents are maintained by a current-direction order-parameter phase-slip

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barrier that has no analog in bilayer quantum Hall systems.

Now let us examine the possibility of persistent currents in a bilayer system when we treat the interlayer tunneling non-perturbatively. First we recall that the interlayer current operator is given by $\hat{I} = eNM_0\Delta_t\hat{M}_y/\hbar$, where N is the total number of electrons. Thus, a current-carrying initial state corresponds to the pseudospin lying in the x-y plane with a nonzero y-component and a concurrent field Δ_t along the x-axis. Since the bilayer system has an easy-plane anisotropy, it is clear that in the absence of a bias-voltage the pseudospin must relax and point along the x-axis or, equivalently, that the initial-state current must decay. In other words, in the absence of bias-voltage, a state with nonzero interlayer current cannot be a steady state. *This physical picture underlies our theory, and provides a transparent way to see why persistent currents are impossible in the presence of a finite interlayer tunneling.*

In a steady state the rate of change of interlayer current is zero. This constraint allows us to express the zero-bias tunnel conductance G_T in terms of a phenomenological lifetime τ for the interlayer current,

$$G_T = \frac{e^2 N M_0 \Delta_t \tau}{\hbar^2}. \tag{1}$$

At a microscopic level, there are several possible channels for the decay of the current; disorder broadening of the quasiparticle bands, local density fluctuations leading to puddles of compressible regions, *etc.* In the present calculation, we assume that the broadening of the mean-field quasiparticle bands because of disorder leads to a finite density of states at the Fermi energy and provides a channel for the current to decay into particle-hole pairs. It is then possible to extract the relaxation time τ for the interlayer current from the dynamical response function $\chi_{yz}(\omega)$,

$$\tau = M_0 \Re[\chi_{yz}^{-1}](\omega \to 0), \tag{2}$$

and obtain the dependence of the zero-bias conductance G_T on interlayer tunneling amplitude Δ_t by using Eq.(1). *Results and Discussion:* We evaluate the response function χ_{yz} using disorder vertex corrections and the generalized random phase approximation, which capture the physics of collective excitations in the presence of a random disorder potential. A straightforward calculation [8] shows that the zero-bias tunnel conductance is given by the Fermi's Golden rule for the mean-field quasiparticles

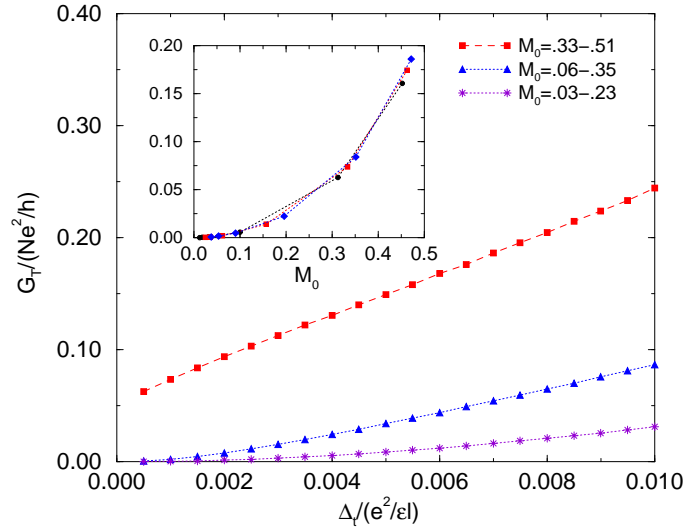$$G_T \propto \Delta_{qp}^2 = (\Delta_t + M_0\Delta_{sb})^2. \tag{3}$$

FIG. 1. Zero-bias conductance G_T as a function of interlayer tunneling amplitude Δ_t for various disorder strengths. The strength of disorder is characterized by the suppression of the order parameter M_0 from its clean limit value $M_0 = 1$. The inset shows the $\Delta_t \to 0$ limit of G_T as a function of the order parameter M_0.

Figure 1 shows the results for the zero-bias conductance G_T as a function of interlayer tunneling amplitude Δ_t for various disorder strengths. We characterize the strength of disorder by the suppression of the mean-field order parameter M_0 from its clean limit value, $M_0 = 1$. When the system is barely phase-coherent ($M_0 << 1$) we find that $G_T \propto \Delta_t^2$; however, as the interlayer phase-coherence develops, the conductance G_T remains finite as $\Delta_t \to 0$.

This result is clearly non-perturbative in the tunneling amplitude Δ_t, and consistent with the breakdown of Fermi's Golden rule for the bare electrons.

The inset in Fig. 1 shows the dependence of the zero-tunneling-amplitude conductance $G_T(\Delta_t \to 0)$ on the order-parameter M_0. We see that the quadratic dependence is consistent with the general result, Eq.(3).

Thus, we find that the zero-bias conductance of a bilayer system is finite and proportional to the square of the quasiparticle tunneling amplitude. This result provides further support for the idea that a weak-coupling Hartree-Fock description of the phase-coherent state is indeed reliable.

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